

Dirichlet Branes of the Covariant Open Supermembrane in $\text{AdS}_4 \times \text{S}^7$ and $\text{AdS}_7 \times \text{S}^4$

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Abstract

We discuss an open supermembrane theory in the $\text{AdS}_4 \times S^7$ and $\text{AdS}_7 \times S^4$ backgrounds. The possible Dirichlet branes of an open supermembrane are classified by analyzing the covariant Wess-Zumino term. All of the allowed configurations are related to those on the pp-wave background via the Penrose limit.

Keywords: open supermembrane, Wess-Zumino term, pp-wave

1 Introduction

Supermembrane theory [1,2] is an interesting subject to study in connection with the M-theory formulation [3]. Open supermembrane theory is also closely related to the Horava-Witten theory [4]. The Dirichlet p -branes of an open supermembrane in flat space was discussed in Refs. [5,6], where it was shown that only the values $p = 1, 5$ and 9 are allowed. The $p = 5$ case corresponds to the M5-brane. The 9-brane implies the existence of the end-of-the world 9-brane as a boundary of the open supermembrane. On the other hand, it is an interesting problem to consider boundary surfaces of open supermembranes on the other backgrounds. We can take classical solutions of supergravities as consistent backgrounds for supermembranes [1]. That is, the kappa-invariance is ensured. In the recent progress, D-branes of the type IIB string on the pp-wave background were studied in the covariant formulation [7] by using the method of Lambert and West [8] (The same method was applied to the type IIA string theory on a pp-wave [9] in Ref. [10]). Moreover, we investigated the boundary surfaces of an open supermembrane on the pp-wave background* in our previous work [16] by combining the method in Ref. [8] with that in [5,6], and we looked deeper into Dirichlet branes of an open supermembrane on the pp-wave background more than in Ref. [14]. In particular, we found the $1/2$ BPS configurations of 1-branes, M5-branes and 9-branes sitting at and outside the origin. The 9-brane configurations sitting at the origin are intimately related to the heterotic matrix model on a pp-wave [17].

In this paper we continue the study of Dirichlet branes of an open supermembrane and consider the case of the open supermembrane in the $\text{AdS}_{4/7} \times S^{7/4}$ backgrounds. By analyzing the covariant supermembrane actions, the possible configurations of boundary surfaces are classified. As the result, we find the configurations of 1-branes, M5-branes and 9-branes preserving a half of supersymmetries at the origin. The $\text{AdS}_{4/7} \times S^{7/4}$ geometries are reduced to the Kowalski-Glikman solution [11] via a certain Penrose limit [18] as shown in [19]. Those are indeed related to the Dirichlet branes on the pp-wave background via the Penrose limit and are consistent to our previous results [16].

This paper is organized as follows: Section 2 is devoted to the setup for our study. We introduce the supermembrane action in the $\text{AdS}_{4/7} \times S^{7/4}$. In section 3, we investigate the Wess-Zumino term in the covariant open supermembrane theory and classify the possible configurations of Dirichlet branes of the membrane sitting at and outside the origin. We also

*The matrix model on the maximally supersymmetric pp-wave background [11] was proposed by Berenstein, Maldacena and Nastase in Ref. [12]. This matrix model is closely related to the supermembrane theory on the pp-wave background [13–15].

discuss the relation between the Dirichlet branes in the $\text{AdS}_{4/7} \times S^{7/4}$ and those on the pp-wave background. Section 4 is devoted to a conclusion and discussions.

2 Covariant Action of Supermembrane on $\text{AdS}_4 \times S^7$ and $\text{AdS}_7 \times S^4$

In this section, we will introduce the action of the supermembrane theory on the $\text{AdS}_4 \times S^7$ and $\text{AdS}_7 \times S^4$ background, and briefly review its relevant aspects to our consideration. We obtain the covariant representation of the Wess-Zumino (WZ) term up to and including fourth order in the $SO(10, 1)$ spinor θ , which will be analyzed in the next section.

The Lagrangian of the supermembrane [1, 2] is formally given as a sum of the Nambu-Goto type Lagrangian \mathcal{L}_0 and WZ term \mathcal{L}_{WZ}^\dagger

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{WZ}, \quad \mathcal{L}_0 = -\sqrt{-g(X, \theta)}, \quad \mathcal{L}_{WZ} = B \quad (2.1)$$

where the induced metric g_{ij} is the pull-back of the background metric G_{MN}

$$g_{ij} = E_i^M E_j^N G_{MN} = E_i^A E_j^B \eta_{AB}, \quad g = \det g_{ij}. \quad (2.2)$$

and the three-form B is defined by $H = dB$ with H being the pull-back of the four-form gauge superfield strength on the supergravity background. The term \mathcal{L}_0 is manifestly spacetime superinvariant while \mathcal{L}_{WZ} is quasi-superinvariant (i.e., superinvariant up to a surface term). The κ -invariance of the action must be imposed in order to match the fermionic and bosonic degrees of freedom on the world-volume. It is known that the condition for the κ -invariance of the action is equivalent to the supergravity equation of motion. For an open supermembrane, the κ -variation leads to a surface term, which must be deleted by imposing appropriate boundary conditions on the boundary of the open supermembrane.

We shall consider supermembranes on the $\text{AdS}_4 \times S^7$ and $\text{AdS}_7 \times S^4$ solutions of the eleven dimensional supergravity. The supervielbeins of the $\text{AdS}_{4/7} \times S^{7/4}$ backgrounds are [20, 21]

$$\begin{aligned} E^A &= dX^M e_M^A - i\bar{\theta}\Gamma^A \left(\frac{2}{\mathcal{M}} \sinh \frac{\mathcal{M}}{2} \right)^2 D\theta, \quad E^{\bar{\alpha}} = \left(\frac{\sinh \mathcal{M}}{\mathcal{M}} D\theta \right)^{\bar{\alpha}}, \\ i\mathcal{M}^2 &= 2(T_A^{B_1 \dots B_4} \theta) F_{B_1 \dots B_4} (\bar{\theta} \Gamma^A) \\ &\quad - \frac{1}{288} (\Gamma_{A_1 A_2} \theta) [\bar{\theta} (\Gamma^{A_1 A_2 B_1 \dots B_4} F_{B_1 \dots B_4} + 24 \Gamma_{B_1 B_2} F^{A_1 A_2 B_1 B_2})], \\ (D\theta)^{\bar{\alpha}} &\equiv d\theta^{\bar{\alpha}} + e^A (T_A^{B_1 \dots B_4} \theta)^{\bar{\alpha}} F_{B_1 \dots B_4} - \frac{1}{4} \omega^{A_1 A_2} (\Gamma_{A_1 A_2} \theta)^{\bar{\alpha}}, \\ T_A^{B_1 \dots B_4} &= \frac{1}{288} (\Gamma_A^{B_1 \dots B_4} - 8 \delta_A^{[B_1} \Gamma^{B_2 B_3 B_4]}), \quad e^A = dX^M e_M^A, \\ F_{\mu_1 \dots \mu_4} &= 6f(\det e_M^A) \epsilon_{\mu_1 \dots \mu_4}. \end{aligned} \quad (2.3)$$

[†]Our notation and convention are summarized in Appendix.

When f is pure imaginary (real), we are dealing with $\text{AdS}_4 \times S^7$ ($\text{AdS}_7 \times S^4$). If we take $f \rightarrow 0$ limit, we can recover the flat Minkowski spacetime. The vielbeins e^A and spin connections ω^{AB} of the $\text{AdS}_{4/7} \times S^{7/4}$ backgrounds can be computed by parameterizing the group manifold corresponding to the algebra given in [20] as $g = e^{X^a P_a + X^{a'} P_{a'}}$ and defining Cartan one-forms as

$$g^{-1}dg = e^a P_a + e^{a'} P_{a'} + \frac{1}{2}\omega^{ab}J_{ab} + \frac{1}{2}\omega^{a'b'}J_{a'b'}, \quad (2.4)$$

$$e^A : e^a = \left(\frac{\sinh Y}{Y}\right)^a_b dX^b, \quad e^{a'} = \left(\frac{\sinh Y'}{Y'}\right)^{a'}_{b'} dX^{b'}, \quad (2.5)$$

$$\omega^{AB} : \omega^{ab} = 4f^2 X^{[a} \left(\frac{2}{Y} \sinh \frac{Y}{2}\right)^{b]}_c dX^c, \quad (2.6)$$

$$\omega^{a'b'} = -f^2 X^{[a'} \left(\frac{2}{Y'} \sinh \frac{Y'}{2}\right)^{b']}_{c'} dX^{c'}, \quad (2.7)$$

where

$$(Y^2)^a_b = \sqrt{|4f^2|}((X^c)^2 \delta_b^a - X^a X_b), \quad (Y'^2)^{a'}_{b'} = -\sqrt{|f^2|}((X^{c'})^2 \delta_{b'}^{a'} - X^{a'} X_{b'}). \quad (2.8)$$

The metric of the background in this parameterization is obtained as $ds^2 = \eta_{AB} e^A e^B$.

On the other hand, the third rank tensor B of $\text{AdS}_{4/7} \times S^{7/4}$ is given by

$$B = B^{(1)} + B^{(2)}, \quad (2.9)$$

$$B^{(1)} = \frac{1}{6} e^{A_1} \wedge e^{A_2} \wedge e^{A_3} C_{A_1 A_2 A_3}, \quad (2.10)$$

$$B^{(2)} = i \int_0^1 dt \bar{\theta} \Gamma_{AB} E(x, t\theta) \wedge E^A(x, t\theta) \wedge E^B(x, t\theta), \quad (2.11)$$

where $E^A(x, t\theta)$ and $E^{\bar{A}}(x, t\theta)$ is obtained by shifting $\theta \rightarrow t\theta$ in E^A and $E^{\bar{A}}$. The second WZ term $B^{(2)}$ becomes

$$\begin{aligned} B^{(2)} &= \frac{i}{2} \bar{\theta} \Gamma_{AB} D\theta \wedge dX^M e_M^A \wedge dX^N e_N^B + \frac{1}{2} \bar{\theta} \Gamma_{AB} D\theta \wedge \bar{\theta} \Gamma^A D\theta \wedge dX^M e_M^B \\ &\quad + \frac{i}{24} \bar{\theta} \Gamma_{AB} \mathcal{M}^2 D\theta \wedge dX^M e_M^A \wedge dX^N e_N^B + \mathcal{O}(\theta^6). \end{aligned} \quad (2.12)$$

We do not need to expand the first term $B^{(1)}$ in the WZ term and the Nambu-Goto part \mathcal{L}_0 with respect to θ , because these terms turn out to be irrelevant in our analysis. The covariant WZ term constructed above will be used in the next section for the classification of boundary surfaces of an open supermembrane on $\text{AdS}_{4/7} \times S^{7/4}$.

3 Classification of Supersymmetric Dirichlet Branes

Here we will classify possible supersymmetric Dirichlet branes of an open supermembrane on the $\text{AdS}_{4/7} \times S^{7/4}$ background by using the covariant action. In conclusion, we will find supersymmetric 1-branes, M5-branes and 9-brane sitting at the origin as Dirichlet branes of an open supermembrane on $\text{AdS}_{4/7} \times S^{7/4}$.

3.1 Boundary Conditions of Open Supermembrane

To begin with, let us introduce the boundary condition on the boundary $\partial\Sigma$ of the world-volume Σ of an open supermembrane. The coordinates should be distinguished in terms of boundary conditions, and so we classify the coordinates of a Dirichlet p -brane as follows:

$$\begin{cases} \bar{A}_i \quad (i = 0, \dots, p) & : \text{Neumann directions} \quad \partial_{\mathbf{n}} X^{\bar{A}} \equiv \partial_{\mathbf{n}} X^M e_M^{\bar{A}} = 0 \\ \underline{A}_j \quad (j = p+1, \dots, 10) & : \text{Dirichlet directions} \quad \partial_{\mathbf{t}} X^{\underline{A}} \equiv \partial_{\mathbf{t}} X^M e_M^{\underline{A}} = 0 \end{cases}, \quad (3.1)$$

where we have introduced a normal vector n^a to the boundary $\partial\Sigma$, and the normal and tangential derivatives on the boundary defined by

$$\partial_{\mathbf{n}} \equiv n^a \partial_a, \quad \partial_{\mathbf{t}} \equiv \epsilon^{ab} n_a \partial_b \quad (a, b = 1, 2). \quad (3.2)$$

Now we shall impose the boundary condition on the fermionic variable θ :

$$P^- \theta|_{\partial\Sigma} = 0 \quad \text{or} \quad P^+ \theta|_{\partial\Sigma} = 0, \quad (3.3)$$

by using the projection operators P^\pm defined as

$$P^\pm \equiv \frac{1}{2} (\mathbb{I} \pm s M^{10-p}), \quad M^{10-p} \equiv \Gamma^{\underline{A}_{p+1}} \Gamma^{\underline{A}_{p+2}} \dots \Gamma^{\underline{A}_{10}}. \quad (3.4)$$

We choose $s = 1$ when the time-direction X^0 is a Neumann direction, while $s = i$ when it is a Dirichlet direction. The requirement that P^\pm are projection operators restricts the value p to $p = 1, 2, 5, 6$ and 9 .

From now on, we examine the κ -invariance of the action (2.1) on $\text{AdS}_{4/7} \times S^{7/4}$. Under the κ -variation

$$\delta_\kappa E^A = 0 \quad \rightarrow \quad \delta_\kappa X^M = i\bar{\theta} \Gamma^M \delta_\kappa \theta + \mathcal{O}(\theta^4), \quad (3.5)$$

the action leads to a surface term only because non-surface terms vanish for the supergravity solution. The kinetic term \mathcal{L}_0 does not contribute to the surface term. The variation of \mathcal{L}_0 includes $\delta_\kappa E_i^A = \partial_i(\delta_\kappa X^{\hat{M}}) E_{\hat{M}}^A$ with $\hat{M} = (M, \alpha)$, but the surface term vanishes because

$\delta_\kappa X^{\hat{M}} E_{\hat{M}}^A = \delta_\kappa E^A = 0$. On the other hand, we can take the vanishing surface term of the variation of $B^{(1)}$ by having additional degrees of freedom at the boundary. It is thus enough to investigate the surface term of the κ -variation of the Wess-Zumino term $B^{(2)}$.

It is convenient to divide the action $S_{WZ}^{(2)} = \int B^{(2)}$ into the following three parts up to and including fourth order in θ

$$S^f = \int d^3\sigma \epsilon^{ijk} \left[\frac{i}{2} \bar{\theta} \Gamma_{AB} (\partial_i \theta + \partial_i X^C T_C^{a_1 \dots a_4} \theta F_{a_1 \dots a_4}) \partial_j X^A \partial_k X^B \right. \\ \left. + \frac{1}{2} \bar{\theta} \Gamma_{AB} (\partial_i \theta + \partial_i X^C T_C^{a_1 \dots a_4} \theta F_{a_1 \dots a_4}) \bar{\theta} \Gamma^A (\partial_j \theta + \partial_j X^D T_D^{a_1 \dots a_4} \theta F_{a_1 \dots a_4}) \partial_k X^B \right], \quad (3.6)$$

$$S^{spin} = -\frac{1}{4} \int d^3\sigma \epsilon^{ijk} \left[\frac{i}{2} \bar{\theta} \Gamma_{AB} \Gamma_{D_1 D_2} \theta \omega_C^{D_1 D_2} \partial_i X^C \right. \\ \left. + \frac{1}{2} \bar{\theta} \Gamma_{CA} \Gamma_{D_1 D_2} \theta \omega_B^{D_1 D_2} \bar{\theta} \Gamma^C (\partial_i \theta + T_E^{a_1 \dots a_4} \theta F_{a_1 \dots a_4} \partial_i X^E) \right. \\ \left. - \frac{1}{2} \bar{\theta} \Gamma_{CA} (\partial_i \theta + T_D^{a_1 \dots a_4} \theta F_{a_1 \dots a_4} \partial_i X^D) \bar{\theta} \Gamma^C \Gamma_{E_1 E_2} \theta \omega_B^{E_1 E_2} \right. \\ \left. - \frac{1}{8} \bar{\theta} \Gamma_{CA} \Gamma_{D_1 D_2} \theta \omega_B^{D_1 D_2} \bar{\theta} \Gamma^C \Gamma_{E_1 E_2} \theta \omega_F^{E_1 E_2} \partial_i X^F \right] \partial_j X^A \partial_k X^B, \quad (3.7)$$

$$S^{\mathcal{M}} = \frac{i}{24} \int d^3\sigma \epsilon^{ijk} \bar{\theta} \Gamma_{AB} \mathcal{M}^2 D_i \theta \partial_j X^A \partial_k X^B. \quad (3.8)$$

The third part $S^{\mathcal{M}}$ contains the \mathcal{M}^2 term, while the second part S^{spin} includes the spin connection. The first part S^f contains all terms other than $S^{\mathcal{M}}$ and S^{spin} .

We analyze these terms in turn. Because the κ -variation boundary terms do not cancel out each other, we examine these terms separately. First, we consider the κ -variation of S^f and study the configuration of Dirichlet branes sitting at the origin. As we will see later, the other terms S^{spin} and $S^{\mathcal{M}}$ do not affect the result at the origin. We next investigate the configuration of Dirichlet branes sitting outside the origin. In this case, the term S^{spin} leads to additional conditions.

3.2 Dirichlet Branes at the Origin

Let us consider the κ -variation of S^f . The surface terms come from the variation of variables with a derivative. Under the variation (3.5), the action leads to surface terms up to and including fourth order in θ

$$\delta_\kappa S^f = - \int d\tau \int_{\partial\Sigma} d\xi \left[i \bar{\theta} \Gamma_{\overline{AB}} \delta_\kappa \theta \dot{X}^{\overline{A}} \partial_{\mathbf{t}} X^{\overline{B}} \right. \quad (3.9)$$

$$+ \frac{1}{2} (\bar{\theta} \Gamma^A \delta_\kappa \theta \bar{\theta} \Gamma_{\overline{AB}} + \bar{\theta} \Gamma_{\overline{AB}} \delta_\kappa \theta \bar{\theta} \Gamma^A) (\dot{\theta} \partial_{\mathbf{t}} X^{\overline{B}} - \partial_{\mathbf{t}} \theta \dot{X}^{\overline{B}}) \\ - \frac{1}{2} (\bar{\theta} \Gamma_{\overline{AB}} T_{\overline{C}}^{a_1 \dots a_4} \theta \bar{\theta} \Gamma^{\overline{C}} \delta_\kappa \theta + \bar{\theta} \Gamma_{\overline{CA}} T_{\overline{B}}^{a_1 \dots a_4} \theta \bar{\theta} \Gamma^{\overline{C}} \delta_\kappa \theta \\ \left. + \bar{\theta} \Gamma_{\overline{CA}} \delta_\kappa \theta \bar{\theta} \Gamma^{\overline{C}} T_{\overline{B}}^{a_1 \dots a_4} \theta F_{a_1 \dots a_4} (\dot{X}^{\overline{A}} \partial_{\mathbf{t}} X^{\overline{B}} - \dot{X}^{\overline{B}} \partial_{\mathbf{t}} X^{\overline{A}}) \right], \quad (3.10)$$

where a dot on a variable means the world-volume time derivative ∂_τ of the variable. The first line vanishes under the condition:

$$\bar{\theta}\Gamma_{\overline{AB}}\delta_\kappa\theta = 0, \quad (3.11)$$

under which the second line also vanishes. This condition restricts the value of p to 1, 5 and 9. Thus we have rederived the well-known result in flat space [5, 6]. It should be noted that only the first and second lines survive in the flat limit $f \rightarrow 0$.

In order for the third and fourth lines to vanish, we must impose the additional constraints:

$$\bar{\theta}\Gamma_{\overline{AB}}T_C^{a_1\cdots a_4}\theta = \bar{\theta}\Gamma_{\overline{B}}^CT_B^{a_1\cdots a_4}\theta = 0. \quad (3.12)$$

These conditions are satisfied under one of the following boundary conditions on (a_1, a_2, a_3, a_4) -directions:

- One of (a_1, \dots, a_4) is a Dirichlet direction and other three directions are Neumann directions.
- Three of (a_1, \dots, a_4) are Dirichlet directions and the remaining one direction is a Neumann direction.

That is, the directions in which 9-branes, M5-branes and 1-branes can span are restricted. We shall classify the possible Dirichlet branes of an open supermembrane on $\text{AdS}_{4/7} \times S^{7/4}$ below.

First, let us consider the configurations of 9-branes, which is classified as follows:

- **Classification of 9-branes** (at the origin) $(\sharp D = 1 \text{ and } \sharp N = 10)$

1. (3, 7)-brane; $M^1 = \Gamma^a$.

The notation of (3, 7)-brane means a brane with the world-volume spanned along three directions of AdS_4 (S^4) and seven directions of S^7 (AdS_7) for $\text{AdS}_4 \times S^7$ ($\text{AdS}_7 \times S^4$). The spanned directions satisfy the Neumann conditions, and the other directions are the Dirichlet ones which are set to zero in the present case.

Next, we shall classify the configuration of M5-branes preserving a half of supersymmetries at the origin. The following two types of M5-brane configurations are allowed.

- **Classification of M5-branes** (at the origin) $(\sharp D = 5 \text{ and } \sharp N = 6)$

1. (3, 3)-brane; $M^5 = \Gamma^{aa'_1\cdots a'_4}$,
2. (1, 5)-brane; $M^5 = \Gamma^{a_1\cdots a_3a'_1a'_2}$.

	$p = 9$	$p = 5$	$p = 1$
M^{10-p}	Γ^a	$\Gamma^{aa'_1 \dots a'_4}, \Gamma^{a_1 a_2 a_3 a'_1 a'_2}$	$\Gamma^{a_1 a_2 a_3 a'_1 \dots a'_6}$

Table 1: Classification of 1/2 BPS Dirichlet branes sitting at the origin.

In the end, we consider the 1/2 BPS configurations of 1-branes at the origin. The possible configurations of 1-brane are given as follows:

- **Classification of 1-branes** (at the origin) ($\sharp D = 9$ and $\sharp N = 2$)

1. (1, 1)-brane; $M^9 = \Gamma^{a_1 a_2 a_3 a'_1 \dots a'_6}$.

Finally, we shall summarize the above result in Tab.1. We would like to remark again that the Dirichlet branes sitting at the origin has been completely classified, since, as explained below, the $S_{\mathcal{M}}$ and S^{spin} have no effect on the above result at the origin. Moreover, it should be noted that our result agrees with the possible AdS embedding.

3.3 Classification of Dirichlet Branes outside the Origin

Here we will consider the contribution of the spin connection. The variation of S^{spin} is given by

$$\begin{aligned} \delta_\kappa S^{spin} = & \frac{1}{4} \int d\tau \int_{\partial\Sigma} d\xi \left[-\frac{1}{2} \bar{\theta} \Gamma_{\overline{AB}} \Gamma_{D_1 D_2} \theta \cdot \bar{\theta} \Gamma^{\overline{C}} \delta_\kappa \theta \omega_{\overline{C}}^{D_1 D_2} - \frac{1}{2} \bar{\theta} \Gamma_{\overline{CA}} \Gamma_{D_1 D_2} \theta \cdot \bar{\theta} \Gamma^{\overline{C}} \delta_\kappa \theta \omega_{\overline{B}}^{D_1 D_2} \right. \\ & \left. - \frac{1}{2} \bar{\theta} \Gamma_{\underline{CA}} \delta_\kappa \theta \cdot \bar{\theta} \Gamma^{\underline{C}} \Gamma_{D_1 D_2} \theta \omega_{\overline{B}}^{D_1 D_2} \right] (\dot{X}^{\overline{A}} \partial_t X^{\overline{B}} - \dot{X}^{\overline{B}} \partial_t X^{\overline{A}}) + \mathcal{O}(\theta^6), \end{aligned} \quad (3.13)$$

where the expression of the spin connection $\omega_C^{AB} = e_C^M \omega_M^{AB}$ is given by

$$\omega_C^{AB} : \omega_c^{ab} = -4f^2 X^{[a} \left(\frac{2}{Y} \tanh \frac{Y}{2} \right)_c^{b]}, \quad \omega_{c'}^{a'b'} = f^2 X^{[a'} \left(\frac{2}{Y'} \tanh \frac{Y'}{2} \right)_{c'}^{b']}. \quad (3.14)$$

Let us consider the vanishing condition for the surface term. The first two terms vanish when

$$\bar{\theta} \Gamma_{\overline{AB}} \Gamma_{\overline{DE}} \theta X^{\underline{E}} = 0, \quad (3.15)$$

which is trivially satisfied for the branes sitting at the origin $X^{\underline{A}} = 0$. On the other hand, this condition is satisfied only for $p = 1$ in the case of branes sitting outside the origin. The vanishing condition for the third term

$$\bar{\theta} \Gamma^{\underline{C}} \Gamma_{\underline{AD}} \theta X^{\underline{D}} = 0, \quad (3.16)$$

is satisfied for branes sitting at the origin, but allows only the 9-brane configuration if we consider outside the origin. Thus, we have found that there is no solution of the 1/2 BPS Dirichlet branes outside the origin.

No Contribution from the $S^{\mathcal{M}}$

Finally we shall discuss the contribution of $S^{\mathcal{M}}$ to our result. We can easily show that $S^{\mathcal{M}}$ including \mathcal{M}^2 does not change the above classification. The variation of $S^{\mathcal{M}}$ is given by

$$\delta_{\kappa} S^{\mathcal{M}} = - \int d\tau \int_{\partial\Sigma} d\xi \frac{i}{12} \bar{\theta} \Gamma_{\overline{AB}} \mathcal{M}^2 \delta_{\kappa} \theta \cdot \dot{X}^{\overline{A}} \partial_t X^{\overline{B}}, \quad (3.17)$$

and hence, in order for the surface terms to vanish, we need to impose the additional condition:

$$\bar{\theta} \Gamma_{\overline{AB}} \mathcal{M}^2 \delta_{\kappa} \theta = 0. \quad (3.18)$$

By the use of the expression of \mathcal{M}^2 , this condition can be rewritten as

$$\begin{aligned} \left(2\bar{\theta} \Gamma_{\overline{AB}} T_{\overline{C}}^{a_1 \dots a_4} \theta \cdot \bar{\theta} \Gamma^{\overline{C}} \delta_{\kappa} \theta - \frac{1}{288} \bar{\theta} \Gamma_{\overline{AB}} \Gamma_{CD} \theta \cdot \bar{\theta} \Gamma^{CD a_1 a_2 a_3 a_4} \delta_{\kappa} \theta \right. \\ \left. - \frac{24}{288} \bar{\theta} \Gamma_{\overline{AB}} \Gamma^{a_1 a_2} \theta \cdot \bar{\theta} \Gamma^{a_3 a_4} \delta_{\kappa} \theta \right) F_{a_1 \dots a_4} = 0. \end{aligned} \quad (3.19)$$

The condition (3.19) is satisfied for the configurations in Tab. 1, as we can easily check. Because there is no nontrivial 1/2 BPS configurations of Dirichlet branes sitting outside the origin, we do not worry about the outside origin case. Therefore we now can say that the Dirichlet branes have been completely classified.

In this paper we have considered the κ -variation boundary terms up to and including fourth order in θ . We expect that our classification of Dirichlet branes is true even at the higher order in θ . In fact, there are some arguments [7] which support that this is the case.

3.4 Penrose Limit of Dirichlet Branes in the AdS Background

Now we will consider the Penrose limit of the Dirichlet branes of the open supermembrane in the $\text{AdS}_{4/7} \times S^{7/4}$ backgrounds. By the use of the Penrose limit, the $\text{AdS}_{4/7} \times S^{7/4}$ geometries are reduced to the maximally supersymmetric pp-wave background, which is often called Kowalski-Glikman (KG) solution. The KG solution can be obtained from $\text{AdS}_{4/7} \times S^{7/4}$ as the Penrose limit only if one makes the light cone coordinates X^{\pm} by taking X^0 and X^{10} from $\text{AdS}_{4/7}$ and $S^{7/4}$, respectively. If we choose both coordinates X^0 and X^{10} from AdS space or sphere, then the Penrose limit is trivial and we obtain the eleven-dimensional flat Minkowski spacetime [19]. In this case, as a matter of course, all kinds of 9-branes, M5-branes and 1-branes are allowed.

Now we will briefly explain how to take the Penrose limit. To begin with, we define the light-cone projection operators as $\mathcal{P}_{\pm} = -\frac{1}{2} \Gamma_{\pm} \Gamma_{\mp}$ and decompose the original spinor θ as $\theta = 1 \cdot \theta = -\frac{1}{2} (\Gamma_+ \Gamma_- + \Gamma_- \Gamma_+) \theta = \theta_+ + \theta_-$. After rescaling the fermionic coordinates $\theta_{\pm} \equiv \mathcal{P}_{\pm} \theta$ as $\theta_- \rightarrow \Omega \theta_-$ and $\theta_+ \rightarrow \theta_+$, we take the limit $\Omega \rightarrow 0$. This is the definition of the Penrose limit in our case.

We will study the Penrose limit of each type of the possible Dirichlet branes at the origin in the AdS coordinate system. For simplicity, we examine the boundary condition $P^-\theta|_{\partial\Sigma} = 0$ below, but the Penrose limit of the condition $P^+\theta|_{\partial\Sigma} = 0$ leads to the similar result.

Penrose Limit of 9-branes

Now let us examine the Penrose limit of 9-brane configurations which is the $(3, 7)$ -brane. The boundary condition is given by

$$s\Gamma^a\theta = \theta. \quad (3.20)$$

When one of 1, 2, 3 is the Dirichlet direction, the condition (3.20) reduces to $\Gamma^I\theta = \theta$, $I = 1, 2, 3$, under the Penrose limit. This is nothing but the boundary condition for $(+, -; 2, 6)$ -brane with $M^1 = \Gamma^I$, which is the possible configuration of 9-branes in the pp-wave case [16]. Note that the resulting pp-wave background after the Penrose limit is not described in the Brinkmann coordinates, but the result of Ref. [16] is written in terms of the Brinkmann coordinates. Therefore we need to take account of the coordinate transformation between two coordinate systems. As we will see in Appendix, the coordinate transformation does not affect our result in this paper.

On the other hand, when the 0-direction (10-direction) in the coordinate system of the $\text{AdS}_4 \times S^7$ ($\text{AdS}_7 \times S^4$) is the Dirichlet direction, then $\theta_+ = 0$ is obtained by the Penrose limit. This means $\Gamma^{-+}\theta = \theta$, or equivalently $\Gamma^{12\cdots 9}\theta = \theta$. Because p is restricted to be 1, 5 and 9, the matrix M^{10-p} cannot be Γ^{-+} . Thus we find $M^9 = \Gamma^{12\cdots 9}$, which is the $(+-)$ -brane boundary condition. These two types of the resulting branes are exactly identical with those found in [16].

Penrose Limit of M5-branes

In the same manner used in the study of 9-branes, the boundary conditions for the M5-branes are modified in the Penrose limit according to the boundary conditions for the 0 and 10-directions.

We can consider the $(3, 3)$ - and $(1, 5)$ -type M5-branes at the origin. First, the boundary condition for $(3, 3)$ -brane, $\Gamma^{aa'\cdots a'_4}\theta = \theta$, leads to

$$\begin{aligned} \Gamma^{II'_1\cdots I'_4}\theta &= \theta && \text{when } 0, 10 \in \text{Neumann}, \\ \Gamma^{+-I'_1\cdots I'_3}\theta &= \theta && \text{when } 0, 10 \in \text{Dirichlet}, \\ \Gamma^{1\cdots 9}\theta &= \theta && \text{otherwise.} \end{aligned} \quad (3.21)$$

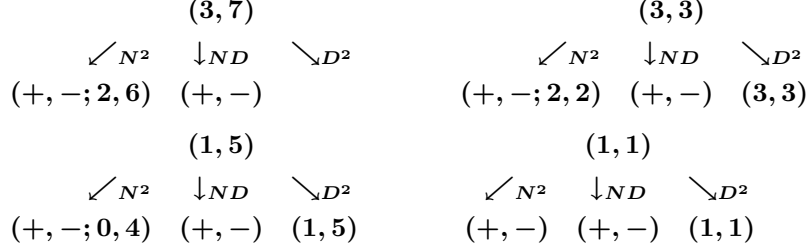


Figure 1: Relation of Dirichlet branes in the $\text{AdS}_{4/7} \times S^{7/4}$ and pp-wave background: Dirichlet branes in the $\text{AdS}_{4/7} \times S^{7/4}$ are mapped to those on the pp-wave background as shown above. Notably, some 9-branes and M5-branes are mapped to the 1-branes expanding in $(+, -)$ -directions on the pp-wave background, when we choose a Neumann coordinate and a Dirichlet one in order to make the light-cone directions.

The last condition follows because in this case the boundary condition becomes $\theta_+ = 0$ by taking the Penrose limit. These are the boundary conditions for $(+, -; 2, 2)$ -, $(3, 3)$ - and $(+, -)$ -branes respectively. Secondly, the boundary condition for $(1, 5)$ -brane, $s\Gamma^{a_1 \dots a_3 a' b'} \theta = \theta$, becomes

$$\begin{aligned}
\Gamma^{I_1 \dots I_3 I' J'} \theta &= \theta && \text{when } 0, 10 \in \text{Neumann}, \\
i\Gamma^{+-IJ I'} \theta &= \theta && \text{when } 0, 10 \in \text{Dirichlet}, \\
\Gamma^{1 \dots 9} \theta &= \theta && \text{otherwise.}
\end{aligned} \tag{3.22}$$

These are $(+, -; 0, 4)$ -, $(1, 5)$ - and $(+, -)$ -branes, respectively. All of branes obtained in the Penrose limit are contained in the list of the classification in the case of pp-wave [16].

Penrose Limit of 1-branes

In this case, the boundary condition, $s\Gamma^{a_1 \dots a_3 a'_1 \dots a'_6} \theta = \theta$, reduces to

$$\begin{aligned}
i\Gamma^{+-IJ I'_1 \dots I'_5} \theta &= \theta && \text{when } 0, 10 \in \text{Dirichlet}, \\
\Gamma^{1 \dots 9} \theta &= \theta && \text{otherwise,}
\end{aligned} \tag{3.23}$$

which correspond to $(1, 1)$ - and $(+, -)$ -branes, respectively. Again, these are contained in the classification in [16].

As a summary, we depict in Fig. 1 the relation of branes in the AdS background to those in the pp-wave. We should remark that all of the possible D-brane configurations in the pp-wave case have been recovered as the Penrose limit of those in the AdS case. Moreover, our result is consistent to the Penrose limit of the embedded AdS brane in eleven dimensions [22].

Finally, we would like to comment on the Penrose limit of the Dirichlet branes outside the origin. As a matter of course, we can find the 1/4 BPS Dirichlet branes even outside the origin by imposing a further boundary condition on the spinor θ , and some of such configurations would be mapped to the 1/4 BPS or 1/2 BPS 1-branes outside the origin on the pp-wave background by taking the Penrose limit.

4 Conclusion and Discussion

In this paper we have discussed the boundary surfaces (Dirichlet branes) of an open supermembrane in the $\text{AdS}_{4/7} \times S^{7/4}$ backgrounds. By analyzing the covariant Wess-Zumino term, we have classified the allowed configurations of Dirichlet branes at the origin. All of 1/2 BPS configurations are mapped to those preserving a half of supersymmetries on the pp-wave background obtained in [16] by taking the Penrose limit. In particular, when we choose a Neumann direction and a Dirichlet one to form the light-cone directions, the 9-branes and M5-branes are changed into the 1-branes spanning in the $(+, -)$ -directions.

In addition, we have discussed the boundary surfaces outside the origin, and it was found that the 1/2 BPS configurations are not allowed. As a matter of course, it would be possible to find less supersymmetric Dirichlet brane configurations, such as 1/4 BPS objects. This is an interesting future problem. It is also important to clarify the preserved supersymmetries in the Penrose limit for these Dirichlet brane configurations in the $\text{AdS}_{4/7} \times S^{7/4}$ background.

It is one of the most interesting problems to consider the application of our results for the study of the $\text{AdS}_{4/7} \times S^{7/4}$ correspondence, and to clarify the field contents in the dual gauge theory. Similar analysis with this paper is applicable for the AdS string case, and we can classify the possible configurations of D-branes of the AdS string, and discuss their Penrose limits using [23]. If such a study could succeed, then we possibly might study the AdS/CFT correspondence at the string theoretical level without using the Penrose limit as far as concerns D-branes of the AdS string. We will study in this direction as a future work [24].

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Appendix

Notation and Convention

In this place we will summarize miscellaneous notation and convention used in this paper.

Notation of Coordinates

For the supermembrane in the eleven-dimensional curved space-time, we use the following notation of supercoordinates for its superspace:

$$(X^M, \theta^\alpha), \quad M = (\mu, \mu'), \quad \mu \in AdS_4(S^4), \quad \mu' \in S^7(AdS_7),$$

and the background metric is expressed by G_{MN} . The coordinates in the Lorentz frame is denoted by

$$(X^A, \theta^{\bar{\alpha}}), \quad A = (a, a'), \quad a = \begin{cases} 0, 1, 2, 3 \\ 10, 1, 2, 3 \end{cases}, \quad a' = \begin{cases} 4, \dots, 9, 10 & \text{for } AdS_4 \times S^7 \\ 0, 4, \dots, 9 & \text{for } AdS_7 \times S^4 \end{cases},$$

and its metric is described by $\eta_{AB} = \text{diag}(-1, +1, \dots, +1)$ with $\eta_{00} = -1$. The light-cone coordinates are defined by $X^\pm \equiv \frac{1}{\sqrt{2}}(X^0 \pm X^{10})$.

The membrane world-volume is three-dimensional and its coordinates are parameterized by $(\tau, \sigma^1, \sigma^2)$. The induced metric on the world-volume is represented by g_{ij} .

SO(10, 1) Clifford Algebra

We denote a 32-component Majorana spinor as θ , and the $SO(10, 1)$ gamma matrices Γ^A 's satisfy the $SO(10, 1)$ Clifford algebra

$$\{\Gamma^A, \Gamma^B\} = 2\eta^{AB}, \quad \{\Gamma^M, \Gamma^N\} = 2G^{MN}, \quad \Gamma^A \equiv e_M^A \Gamma^M, \quad \Gamma^M \equiv e_A^M \Gamma^A,$$

where the light-cone components of the $SO(10, 1)$ gamma matrices are

$$\Gamma^\pm \equiv \frac{1}{\sqrt{2}}(\Gamma^0 \pm \Gamma^{10}), \quad \{\Gamma^+, \Gamma^-\} = -2\mathbb{I}_{32}.$$

We shall choose Γ^A 's such that Γ^0 is anti-hermite matrix and others are hermite matrices. In this choice the relation $(\Gamma^A)^\dagger = \Gamma_0 \Gamma^A \Gamma_0$ is satisfied. The charge conjugation of θ is defined by

$$\theta^c \equiv \mathcal{C} \bar{\theta}^T,$$

where $\bar{\theta}$ is the Dirac conjugation of θ and is defined by $\bar{\theta} \equiv \theta^\dagger \Gamma_0$. The charge conjugation matrix \mathcal{C} satisfies the following relation:

$$(\Gamma^A)^T = -\mathcal{C}^{-1} \Gamma^A \mathcal{C}, \quad \mathcal{C}^T = -\mathcal{C}.$$

For an arbitrary Majorana spinor θ satisfying the Majorana condition $\theta^C = \theta$, we can easily show the formula

$$\bar{\theta} = -\theta^T \mathcal{C}^{-1}.$$

That is, the charge conjugation matrix \mathcal{C} is defined by $\mathcal{C} = \Gamma_0$ in this representation. The Γ^A 's are real matrices (i.e., $(\Gamma^A)^* = \Gamma^A$). We also see that Γ^r ($r = 1, 2, \dots, 9$) and Γ^{10} are symmetric and Γ^0 is skewsymmetric.

Relation to the KG solution

The Kowalski-Glikman solution, which is the maximally supersymmetric pp-wave background in eleven dimensions, can be obtained from $\text{AdS}_{4/7} \times S^{7/4}$ by taking the Penrose limit. Noting that the super-isometry algebra of the KG solution is an Inönü-Wigner contraction of that of $\text{AdS}_{4/7} \times S^{7/4}$ [25], the group manifold for the pp-wave algebra is parameterized by $g = e^{X^+ P_+ + X^- P_- + X^m P_m}$ because we used the parameterization $g = e^{X^a P_a + X^{a'} P_{a'}}$ in (2.5). The metric in this parametrization is written as

$$ds^2 = -2e^+ e^- + (e^I)^2 + (e^{I'})^2,$$

where the vielbeins are given by

$$\begin{aligned} e^+ &= dX^+, \quad z = \frac{\mu}{3} X^+, \quad z' = \frac{\mu}{6} X^+, \\ e^- &= dX^- - \left(1 - \frac{\sin z}{z}\right) \frac{X^I}{(X^+)^2} (X^+ dX^I - X^I dX^+) \\ &\quad - \left(1 - \frac{\sin z'}{z'}\right) \frac{X^{I'}}{(X^+)^2} (X^+ dX^{I'} - X^{I'} dX^+), \\ e^I &= \frac{\sin z}{z} dX^I + \left(1 - \frac{\sin z}{z}\right) \frac{X^I}{X^+} dX^+, \quad e^{I'} = \frac{\sin z'}{z'} dX^{I'} + \left(1 - \frac{\sin z'}{z'}\right) \frac{X^{I'}}{X^+} dX^+. \end{aligned}$$

In order to make the metric to be of the ordinary form in Brinkmann coordinate we change the coordinate (X^\pm, X^m) as

$$\begin{aligned} \hat{X}^+ &\equiv X^+, \quad \hat{X}^I \equiv \frac{\sin z}{z} X^I, \quad \hat{X}^{I'} \equiv \frac{\sin z'}{z'} X^{I'}, \\ \hat{X}^- &\equiv X^- - \frac{\mu}{6} \frac{z - \sin z \cos z}{\sin^2 z} (\hat{X}^I)^2 - \frac{\mu}{12} \frac{z' - \sin z' \cos z'}{\sin^2 z'} (\hat{X}^{I'})^2. \end{aligned}$$

Under this transformation, the metric and the flux become

$$ds^2 = -2d\hat{X}^+d\hat{X}^- - \left[\left(\frac{\mu}{3} \right)^2 (\hat{X}^i)^2 + \left(\frac{\mu}{6} \right)^2 (\hat{X}^{i'})^2 \right] (d\hat{X}^+)^2 + (d\hat{X}^m)^2,$$

and the constant flux $F_{+123} = \mu$, respectively. This reveals the fact that $X^i = 0$ and $X^{i'} = 0$ for $\text{AdS}_{4/7} \times S^{7/4}$ correspond to $\hat{X}^i = 0$ and $\hat{X}^{i'} = 0$ for the KG solution in Brinkmann coordinates. We can easily see that the condition $X^+ = X^- = 0$ corresponds to $\hat{X}^+ = \hat{X}^- = 0$ by the use of the formulae of trigonometrical functions.

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